**Scientific description why a tower cannot collapse from top down**

Abstract:

On September 11, 2001, the *progressive, global, structural collapses* of the WTC 1 and 2 towers at NY were shown live on all five major TV channels in the USA. The towers collapsed from top down when loose top parts displaced down and crushed the bottom parts by gravity producing fountains of debris, dust and broken parts and clouds of smoke shooting out upwards and sideways. In this paper is described how the phenomenon *progressive, global, structural collapse* is physically impossible. Reason is that a top assembly of elements of any structure cannot apply sufficient force on the bottom assembly to destroy the latter. There is not enough energy available. Anyone suggesting something else or shows it ‘live on TV’ is a simple terrorist.

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**Scientific description why a tower cannot collapse from top down**

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It is however very easy to demonstrate theoretically, numerically, in a laboratory and/or in real scale that a human built tower structure $A$ cannot globally collapse by itself from top down, when a small top part $C$ of $A$ drops on and impacts remainder of $A$ by gravity from above. Reason is simply that such a structure is always stronger at bottom than top.

Consider a tower structure of $N$ floors, each of mass $m$, labeled from top ($i = 1$) to bottom ($i = N$).

- Adjacent floors are separated by springs.
• Floor $n$ is supported by $n$ springs.
• The spring ends are fixed to the floors.

All springs are weightless, have rest length $L$ (m), and are perfectly elastic with constant stiffness $k$ (N/m) until compressed longitudinally and laterally a critical longitudinal displacement $x^C$, when they compress plastically another displacement $x^P$, after which they finally fail (and the length becomes $0$).

The springs represent the load carrying elements of the tower structure, e.g. walls, pillars and columns.

The tower is placed in a vacuum on a very large inertial mass, i.e. ground and subjected to a uniform gravitational field directed downwards. $g = 9.82 \text{ m/s}^2$.

**Static equilibrium**

The tower is in static equilibrium if $F_n = nkx = nmg$ (N) → $x = mg/k$. In static equilibrium the displacement $x$ is everywhere the same and the same is true for the strain $\varepsilon = x/L$ and stress $\sigma$ in the spring.

**Safety factor**

The $n$ springs below floor $n$ can each support a load of $F^C = kx^C$ before plastic deformation starts. If the actual design load of a spring is $F$, then the safety factor $S = F^C/F$.

**Elastic and plastic strain energy**

The work performed in compressing a spring elastically is $E = kx^2/2$ (J).

With $x^C = 0.01L$, which is typical for a pillar in a tower structure, a spring can elastically absorb $E^S = 0.00005kL^2$ (J).

If a spring is overloaded elastically and starts to deform plastically, the spring will develop plastic hinges which absorb more energy $E^P$ and which takes a certain time $t^P$ – and its effective length finally becomes $0$ – it has broken!

Let’s assume that $E^P = 2E^S$.

**Numerical example**

Assume that the tower has $N = 110$ floors, each with mass $m = 3.6 \text{ Mkg}$ (i.e. $3600$ tons) and that $L = 3.7$ meter.

It means that the tower is $407$ meter tall, like WTC 1 or 2.

The total number of springs in a tower of $N$ floor is $N(N+1)/2$. 

Thus a tower with 110 floors consists of 6,105 springs. The top floor (or roof) is supported by 1 spring and the bottom floor is supported by 110 springs.

The assembly of the 13 uppermost floors contains 91 springs, while the assembly of 97 lowest floors contains 6,014 springs.

Let’s call the assembly of the 13 uppermost floors top part C and let’s remove the 13 bottom springs from C. C then consists of 13 m connected by 78 springs (1 between top two m, 12 between lowest two m). The 78 springs in C can together absorb $E_C = 78E^S$. Top part C weighs 48,800 tons.

Let’s call the assembly of the 97 lowest floors bottom part A. A’s 110 lowest springs are connected to ground. The 6,014 springs in A can together absorb $E_A = 6,014E^S$. Bottom part A weighs 349,200 tons and is 7.2 times heavier than part C. If energy absorption is a measure of ‘strength’, A is 77.1 times ‘stronger’ than C.

*Spring details - same everywhere*

Static force $F$ in every spring is $F = mg = 35,352$ MN

The spring may have cross area of about 0.5 m² if it is of steel. The static stress in the spring is then about 70 MPa. Note that the static stress is same everywhere in every spring - at top and bottom of the structure.

Let’s assume $S = 3$, i.e. the springs will commence plastic deformation at 210 MPa. The material of the spring is steel.

Note that the total cross area of removed springs below top part C is 6.5 m² (13 springs each 0.5 m²) and that the total cross area of springs at bottom of bottom part A is 55 m² (110 springs each 0.5 m²). The tower structure evidently gets ‘stronger’ with more springs added further down.

The critical force $F_C$ of a spring is $F_C = 106,056$ MN. After that it deforms plastically absorbing more energy and soon has length 0.

Let’s assume that spring stiffness $k = 3$ GN/m that is typical when core and perimeter wall structure of WTC is replaced by one spring (a bundle of steel elements) with cross area 0.5 m² that can deform in 3-D. Actual $k$ is easy to establish in a laboratory.

Then

\[ x^C = 0.037 \text{ m} \]

\[ E^S = \frac{k(x^C)^2}{2} = 2.053 \text{ MJ} \]

\[ x = 0.037/3 \text{ m} \]
When the tower is in static equilibrium, each spring is compressed 0.037/3 meter and 0.228 MJ energy \( E \) is stored in it elastically. Our 110 floors tower is therefore elastically compressed 1.357 meters. However, a spring can be compressed elastically to \( x^C = 0.037 \text{ m} \) (or 1% of \( L \)) before it starts to deform plastically, and the energy \( E^C \) required to compress it is then 2.053 MJ. From an energy absorption point of view, the factor of safety is 9 (actually static \( S^2 \)). All 6105 springs in our tower can elastically absorb 12.53 GJ energy.

Top part \( C \) can absorb elastically totally \( 78 \times 2.053 = 160.1 \text{ MJ energy} \). Bottom part \( A \) can absorb elastically \( 6014 \times 2.053 = 12.35 \text{ GJ energy} \).

By dropping top part \( C \) a certain distance, e.g. \( L \), a certain amount of potential energy \( E^D \) is released, where \( E^D = 13mLg = 1.7 \text{ GJ} \). It is 13.6% of what the tower itself can absorb elastically. By simple structural damage analysis you can establish whether \( C \) can damage \( A \), ground or itself \( C \).

**Experiment 1: Structure part \( C \) collapses from bottom up**

In Experiment 1 is described how something weak, top part \( C \), dropped on rigid ground will be affected by the impact. The top part \( C \) assembly of 13 m is dropped on ground from distance \( L \). At the impact \( C/ground \) total 1.7 GJ is applied to ground and \( C \). The ground does not damp the impact. It is rigid and can absorb plenty energy. Evidently top part \( C \) itself damps the impact - it becomes compressed and damaged:

As top part \( C \) is 44.4 meter tall and consists of 13 m separated by springs, it is the bottom floor \( m \) of \( C \) that physically contacts ground and is arrested by ground at the impact. The remaining 12 m above continue to displace down and compress the springs below. A certain damping takes place, when the springs compress elastically and plastically.

As the dynamic forces acting on top part \( C \) and ground at impact and later are equal and opposite (the dynamic force is the energy applied \( (0.5E^D) \) divided by the displacement of the force - the structure is compressed and maybe damaged), it follows that top part \( C \) will absorb \( 0.5E^D \) and rigid ground will also absorb \( 0.5E^D \) in the impact. It would then appear that 0.85 GJ energy is applied on top part \( C \) one way or another and as top part \( C \) can only absorb 0.16 GJ elastically and 0.32 GJ plastically (or 0.48 GJ totally), all springs in top part \( C \) will fail. Ground is rigid and undamaged and can easily absorb 0.85 GJ.

Top part \( C \) is, as seen, not very strong, and it is why its springs are 100% broken at impact with ground.

In what order will the springs in top part \( C \) fail? It can be seen on videos of controlled demolitions of buildings, where the bottom supports are destroyed first and structure above drops and hits ground that destruction is from bottom up, thus:
The bottom 12 springs fail first at impact with ground, 12 floors \( m \) above then drop down \( L \), more potential energy is released, 11 springs fail at second impact, 11 \( m \) drop down \( L \), 10 springs fail, 10 \( m \) drop, etc, etc, until the last one top spring fails and the last roof \( m \) impacts ground from \( L \). Top part \( C \), 12 \( L \) tall, is destroyed from bottom up in 12 steps that takes a certain time. Total energy released due to failed springs after initial impact \( C/ground \), i.e. when \( C \) is destroyed, is \( 72 \text{mLg} \) or 8.82GJ (it was stored in \( C \) prior impact) and all of it is absorbed by the ground.

You can say that top part \( C \) is **crushed-up** by 13 impacts with solid ground.

Bažant & Co suggest in their ridiculous, peer reviewed papers published in Journal of Engineering Mechanics that weak top part \( C \) is rigid and remains intact, when impacting, e.g. stronger bottom part \( A \), and that is one false idea of Bažant later adopted by NIST as true.

**Experiment 2: Structure \( A \) loaded on top deforms elastically**

In Experiment 2 is shown how bottom part \( A \) behaves, when statically loaded from above. Bottom part \( A \) is thus positioned on ground without \( C \).

\( A \) is, as already described, a tower structure of 97 \( m \) and 97 \( L \) tall. It consists of 6 014 springs that can absorb elastically totally 12.346 GJ energy. 1/9th of this energy is already stored statically in the tower so another 8/9th or 10.974 GJ can be applied and absorbed elastically. Considering plastic deformations another 20 GJ can be absorbed.

The top floor \( m \) of \( A \) is supported by 14 springs located on the second floor \( m \) below that is supported by 15 springs further below, etc.

The 14 top springs of bottom \( A \) can elastically absorb totally 28.74 MJ but are statically only loaded by one \( m \). you can add another 13 \( m \) (i.e. \( C \)) on the 14 springs and then they are under original design load, like all the other springs in \( A \).

Thus, just adding 13 \( m \) on top of \( A \) nothing special will happen except that all springs in \( A \) are again under original design, static load.

What happens if, in lieu of slowly putting 13 \( m \) on \( A \) we drop 13 \( m \) on \( A \) from \( L = 3.7 \) meter and we let 13 \( m \) impact \( A \)! Will the dynamic force at impact crush \( A \)?

**Experiment 3: Bottom part \( A \), impacted on top, damps impact due to elastic and plastic deformations**

In experiment 3 is shown what happens when a rigid mass of 13 \( m \) impacts \( A \) from above. 13 \( m \) are dropped on the top \( m \) of \( A \) from \( L = 3.7 \) meter. The 13 \( m \) are connected together without any interconnecting springs, and that assembly is here called \( D \). \( D \) is one rigid mass of 13 \( m \).
At impact $D/A$ 1.7 GJ is applied to $A$ and $D$ and, as in experiment 1 0.85 GJ is applied to $A$. $A$ can totally absorb 10.974 GJ so you would expect rigid $D$ to bounce on $A$. The dynamic force $F$ applied on $A/D$ is, as stated above, simply the energy applied divided by displacement of force during impact/compression that takes a certain time. The initial impact will be followed by more impacts, if further $m$ gets loose and drops, but energy released in each impact will be elastically and maybe plastically absorbed by intact springs. The beauty of a spring is that it can absorb energy multiple times, when loaded in succession and that this absorption takes time. The spring acts like a shock absorber transmitting energy to other springs and ground.

It is very strange that NIST suggests without any evidence in its 10 000 pages 911-report that little, weak $C$ (or rigid $D$) can apply energy on big, strong $A$ that $A$ cannot absorb! The figures say something completely different! Why does NIST lie and spread false information to the public. Is it in order to support terrorism?

It may be argued that the top 14 top springs and the 15 springs in the next layer of $A$ below may be destroyed locally in overload by the dynamic forces at impact with $D$ and that some extra energy released, when $D$ and one or more loose $m$ displace down $L$.

The 14 top springs of $A$ can totally absorb elastically $14 \times 2.053 = 28.74$ MJ and maybe plastically totally say $86$ MJ and the next 15 springs about the same. The plastic destruction (failure) of springs takes time, so in the mean time the dynamic impact force (i.e. energy divided by displacement) can be absorbed elastically by intermediate springs and transmitted to ground (as a seismic wave). When one layer of springs is destroyed all $m$ above displace down $L$ and more energy is released - a second impact - and has to be absorbed by intact springs like a shock absorber.

So $D$ applies 850 MJ on $A$ and about 172 MJ can be absorbed by destruction of the two top layers of springs in $A$ and the rest is absorbed elastically by 95 other layers of springs in $A$ and transmitted to ground.

That $D$ would destroy all 6 014 springs of bottom part $A$ is unlikely. The springs of $A$ will dampen the impact of $D$ and further loose top $m$ of $A$ dropping, while only some local failures occur close to interface $D/A$.

It is quite easy to verify experiment 3 in a laboratory. Just take the top $C$ of any tower structure, compress it to a rigid block $D$, and drop $D$ on the bottom part $A$ and see what happens. Rigid $D$ will always bounce and stop after producing some local failures at top of $A$, i.e. the weakest part of $A$!

**Experiment 4: Small top $C$ cannot crush a bigger bottom $A$**

In experiment 4 is shown what happens when top part $C$ impacts bottom part $A$ from above. Top part $C$ is thus dropped on bottom part $A$ from $L = 3.7$ m. This is the famous WTC 1 event.
13 top floors \( m \) of WTC 1 drop on 97 intact floors/columns \( m \) below (and according videos of suspect origin the 97 floors/columns below are destroyed in a fountain of smoke, dust and debris – terrible – as shown ‘live on TV’ September 11, 2001! In reality, of course, it cannot happen).

At impact \( C/A \) 0.85GJ is applied to \( C \) (with 12 springs at bottom) and 0.85GJ is applied to \( A \) (with 14 springs at top) as explained above. However, \( C \) does not impact rigid ground as in experiment 1 and \( A \) is not impacted from above by rigid \( D \) as in experiment 3.

In fact only the top \( m \) of \( A \) supported by 14 springs below and the bottom \( m \) of \( C \) supported by 12 springs above contact each other in the impact and the dynamic forces are then transmitted via the springs to other \( m \) in \( A \) and \( C \) via springs that behave elastically and dampen the impact. The impact, like in experiment 3, will be split in sub-impacts when/if further floors \( m \) gets loose and drops but energy released in each sub-impact will be elastically and plastically absorbed by the springs.

So in experiment 4 the initial impact will really be dampened, i.e. take longer time, as both \( A \) and \( C \) ... and ground ... will dampen (absorb the energy of) the local impact \( C/A \). It also means that the dynamic forces are reduced. That small/weak \( C \) will crush big/stronger \( A \) at increasing speed and by gravity is impossible.

That \( C \) - that can absorb much less energy elastically and plastically than \( A \) - can apply, via dynamic forces at impacts, and release, via structural/spring failures, more energy on \( A \) and destroy \( A \) is impossible: \( C \) will destroy its own springs first, before \( A \) is starting to get destroyed and then \( C \) cannot apply or release more energy to destroy \( A \). In reality there will only be some local failures at interface \( C/A \) at impact, \( C \) and \( A \) then get locally entangled, friction develops and \( C \) will then just bounce on top of \( A \). \( A \) arrests \( C \)! There is not enough energy for anything else.

As seen above 0.85 GJ energy is applied to 12 bottom springs in \( C \) (and to remaining 66 springs above) and 0.85 GJ energy to 14 top springs in \( A \) (and to remaining 6 000 springs below) at impact \( C/A \).

What happens if 0.065 GJ energy \( E \) is applied to one spring with stiffness \( k = 3 \) GN/m and 3.7 meter length \( L \)? Answer: the spring will compress \( x = 0.147 \) meter (as \( x^2 = E/k \)) due to the impact or 4% \( L \). As one spring in our example can only elastically compress 1% it means that the spring plastically deforms and breaks at impact. However, our spring is not alone but supported by other springs above and below in the structure so you have to consider that. Evidently the 6 014 springs in \( A \) can easily absorb totally 0.85 GJ energy elastically (as shown in experiment 3). If the 78 springs in \( C \) can do it, is another matter (as shown in experiment 1).

It is quite easy to verify experiment 4 in a laboratory. Just take the top part \( C \) of any tower, and drop it on the bottom part \( A \) and register what happens. My experience is that \( C \) always
bounces on and is arrested by A, but I may be wrong. I have only tested a limited amount of towers. No smoke, dust, debris or ejections were produced when dropping C on A.

Many people believe that scale or size of the structure matters, e.g. that a small (model of a) structure cannot crush itself but that a bigger structure can or that material matters, e.g. that a structure of brittle elements/connections will collapse but not a structure of more ductile elements/connections. However, to believe things like that is unscientific, terrorist nonsense.

Experiment 4 impact, elastic compression of springs and damping of parts can of course easily be modeled mathematically using Finite Element Methods for any size of tower springs structure/elements/connections/material. A linear spring-damper model of the form $f(t) = k*x(t) + c*v(t)$, where $x = \text{input displacement}$, $v = \text{input velocity}$, and $f(t) = \text{output force}$ can be developed based on test data in the time domain of the springs. The term $k$ is the spring stiffness (N/m) and $c$ is the viscous damping coefficient (Ns/m). With $k = 3 \text{ GN/m}$ and $c = 0.3 \text{ GNs/m}$ the tower parts A and C become flexible and will visibly deform, compress, oscillate, be damped, for several seconds after impact C/A. Plastic deformation and it’s time to develop failures of a spring are more complex to model mathematically (but it can be done). That a 407 meter tower structure will explode in smoke, dust and debris, rubble being formed and collapse from top taking place in 15 seconds as shown ‘live on TV’ Tuesday morning 11 September 2001 in the USA is evidently not possible in reality. What was shown ‘live on TV’ was just a stupid movie made by disaster animators Hollywood style! Imagine that!

The writer’s attempts to crush a structure by dropping its top on it have, naturally, always ended up with no springs, elements or connections failing in A and C and only bouncing/arrest of C taking place.

**Conclusion**

The writer has never seen a top part C of a tower structure impacting and destroying the bottom part A due to gravity. Reason is that such destruction is physically impossible! A always arrests C. Anyone suggesting something else or shows it ‘live on TV’ is a simple terrorist.

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